Public Choice (2017) 173:109–130
DOI 10.1007/s11127-017-0466-4

Efficient bilateral taxation of externalities

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Received: 8 March 2017 / Accepted: 27 June 2017 / Published online: 30 June 2017

Abstract In the context of the example of a factory whose smoke emissions affect a nearby laundry, Coase (J Law Econ 3:1–44, 1960) argued for taxing the laundry as well as the factory, while Baumol (Am Econ Rev 62:307–322, 1972) argued for taxing only the factory. Consistent application of marginal cost pricing shows that the efficient tax on laundries is positive when the number of laundries is finite and that the tax approaches zero in the limit as the number of laundries approaches infinity. The efficient tax on factories is bounded away from zero, regardless of the number of factories. Our framework is an application of the Vickrey–Clarke–Groves family of truth-telling mechanisms that require each agent to bear the full social cost of changing the outcome that would have prevailed had she not participated in the decision. Until now, the literature has not fully resolved the discrepancies between Coase's and Baumol's arguments, and even contemporary textbooks on environmental economics and public economics do not offer correct and complete analyses.

Keywords Non-rival bads · Truth-telling mechanisms · Marginal cost pricing · Pigouvian tax

JEL Classifications H21 · H23

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1 Introduction

The idea that people behave efficiently if they bear the marginal costs of their actions goes back at least as far as Dupuit's (1844) paper *On the Measurement of the Utility of Public Works*, becoming part of the common wisdom of economists at the beginning of the twentieth century through the writings of Alfred Marshall, Arthur Pigou and, later, Harold Hotelling. It took until the 1960s for economists to realize that social efficiency requires that all parties to an activity bear the marginal costs of their actions. Still, only now are economists beginning to appreciate the idea fully.

We apply marginal cost pricing to the familiar situation of laundries that operate near smoke-emitting factories. Coase (1960) argued that in situations in which parties could not achieve efficient outcomes through bargaining, there was as much reason to tax the laundries for inhibiting the factories as to tax the factories for harming the laundries. In contrast, Baumol (1972) argued that efficiency does not require taxes on laundries. Neither of them got the argument completely right, and consistent application of marginal cost pricing resolves the problems in their respective arguments. We show that Coase generally is correct as long as the number of laundries is finite, while Baumol is correct in the limit as the number of laundries approaches infinity.

The intuition behind our result is straightforward. When multiple factories and multiple laundries are involved, the introduction of the first emission of a new factory creates harm for laundries at the level at which the benefit of the last unit of prior emissions was exactly offset by the tax on emissions. This cost does not decline as the number of factories increases. On the other hand, a report from a new laundry of incremental harm from emissions has offsetting effects, because, at the initial margin, the costs to factories that must now cut back on their emissions are offset fully by benefits to laundries that now have cleaner air. When a laundry reports more than incremental harm, the factories' costs from reducing their emissions are offset only partially by the benefits from cleaner air to the other laundries, and the laundry that reports more than incremental harm pays the difference. As the number of laundries increases, the share of any one laundry in the benefits of clean air becomes smaller and smaller, and the Pigouvian tax on laundries approaches zero. Because factories' emissions cause only social costs while the costs that laundries impose on factories are offset partly or—in the limit, fully—by benefits to other laundries, it is not surprising that the associated efficient taxes have different limits as the numbers of factories and laundries become indefinitely large.

Our framework is related to an underappreciated truth-telling mechanism in Vickrey (1961) for the valuation of rival goods, which, as Tideman and Plassmann (2017) suggest, can be interpreted as a combination of continuous multi-person versions of Vickrey's second-price auction and the mechanism proposed by Becker et al. (1964). Vickrey's mechanism achieves truthful revelation of each buyer's willingness to pay by charging her the smaller of her revealed willingness to pay and the social cost of each unit she demands. The social cost of a buyer's market participation has two components—first, the opportunity cost of the resources necessary to provide the additional units demanded, and second, the reduction in the consumer surplus of other buyers who buy fewer units when the increase in demand leads to a higher price. Vickrey shows that charging buyers the actual social cost of their demand—rather than charging the cost of the marginal unit for all units demanded—provides buyers with the incentive to reveal their willingness to pay, because they might otherwise forego advantageous trades.[1]

[1] Vickery (1961) describes a corresponding mechanism that offers each seller the larger of her revealed marginal cost and a prospective buyer's willingness to pay; doing so provides sellers with incentives to reveal their true marginal costs because they might otherwise forego advantageous trades.

Our Pigouvian tax on factories is identical to the charge on buyers in Vickrey's mechanism, as each factory pays for the harm that its emissions cause to laundries as well as for costs to other factories from reductions in the value of the opportunities they face because of higher Pigouvian tax schedules. Although Vickrey designed his mechanism for the provision of rival goods, so that it does not apply directly to goods like clean air whose consumption is non-rival, his insight on marginal cost pricing suggests our Pigouvian tax on laundries—the difference between the cost to factories and the benefit to other laundries that result from higher Pigouvian taxes on incremental units of emissions. While Vickrey's application of his mechanism to rival goods led to a budget deficit, our application to a non-rival bad leads to a budget surplus. Our analysis therefore represents a continuous application of the familiar Vickrey–Clarke–Groves (VCG) family of mechanisms that provide each agent with an incentive to reveal her respective marginal benefit or marginal cost, by requiring her to bear the full social cost of changing the outcome that would have prevailed had she not participated.

Consistent application of marginal cost pricing to bilateral taxation in the context of emissions provides all agents with an incentive to reveal truthfully their respective marginal benefit and marginal cost schedules. Marginal benefits and costs generally are unobservable, and governments need to know the combined marginal costs of laundries to determine the appropriate Pigouvian tax on factories. Hence bilateral taxation increases the applicability of the Pigouvian framework when the number of laundries is finite.[2]

In Sects. 2 and 3 we consider situations with convex opportunities and show that the efficient tax on laundries is positive but goes to zero in the aggregate as the number of laundries approaches infinity. We provide a graphical argument in Sect. 2 and the corresponding mathematical argument in Sect. 3. We consider non-convexities in Sect. 4. In Sect. 5, we place our contribution within the sparse literature on bilateral taxation of environmental externalities and examine the arguments made in recent textbooks. We extend the application of bilateral marginal cost pricing to additional topics in Sect. 6 and conclude in Sect. 7.

2 Convex opportunities—graphical argument

Figure 1 shows the case of one factory whose operation leads to a negative externality on one nearby laundry. The horizontal axis in the figure measures the level of emissions, the downward sloping curve *DKC* shows the (net) marginal benefit from emitting that the factory and the consumers of the factory's output obtain, and the upward sloping curve *AKH* shows the social marginal cost of these emissions that is borne by the laundry.[3] The factory emits the amount *C* if it ignores the social marginal cost of its emissions, and it emits the amount *B* if it takes this cost into account. Restricting emissions to *B* rather than leaving them at *C* imposes a cost on the factory in terms of lost profit and on consumers in terms of lost consumer surplus equal to *KBC*. Call this the "control cost" of restricting

[2] Ng (2007) makes a similar argument, although he does not relate the tax on residents to Vickrey's framework. Instead, Ng achieves truthful revelation of marginal damages by restricting the permissible heterogeneity of marginal damages across residents.

[3] This type of diagram is presented in Buchanan and Stubblebine (1962) as well as in Ng (1971, 2007). We assume that the marginal benefit schedule *DKC* is concave rather than convex to make it easier to relate our graphical argument to Fig. 2 in Vickrey (1961). Our argument holds for convex as well as concave marginal benefit schedules.

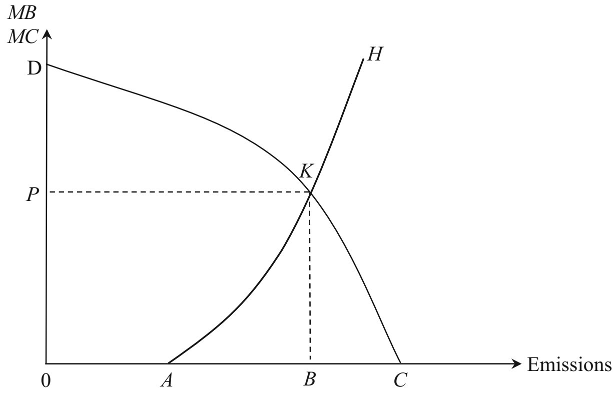


Fig. 1 The situation with one factory and one laundry

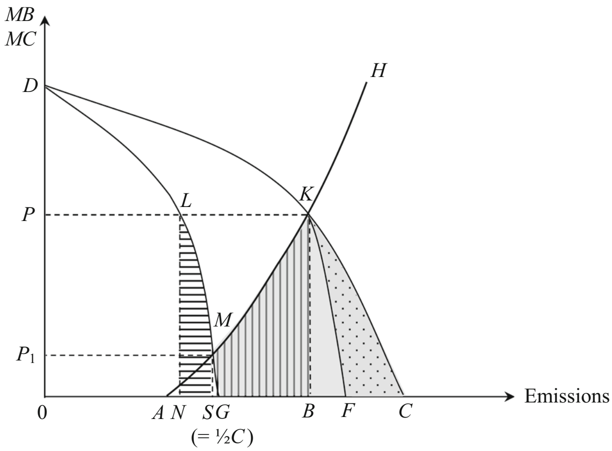


Fig. 2 The marginal cost of adding a second factory when all factories are of equal size

emissions. Restricting emissions to *B* rather than *A* imposes emission costs on the laundry equal to *ABK*.

Coase (1960) argued that providing incentives for efficient behavior requires that both parties be taxed for the costs imposed on the other. A factory that does not bear the laundry's cost has no incentive to reduce emissions below *C*, while a laundry that does not bear the factory's control cost has no incentive to locate elsewhere if it can accomplish this at a lower cost. Thus, efficiency requires that each party bear the full cost *AKC*: the factory pays a tax equal to *AKB*—the cost that it imposes on the laundry by emitting *B*—and bears the control cost *KBC*, while the laundry pays a tax equal to the factory's control cost *KBC* and bears the cost *AKB* of the emissions that still occur after the factory adjusts to its Pigouvian tax. The laundry receives no compensation for the damages *AKB*. The more customary approach would be to identify the marginal cost of emissions (*P*) and charge this amount for all emissions (that is, charge the factory 0PKB). But this is inefficient because this amount exceeds the cost imposed on the laundry, and the additional cost to the factory could cause the factory to shut down when the net benefit from remaining open is positive.

Consider next how taxes change when the number of factories and the number of laundries increase, which amounts to identifying the contribution of an individual factory to total damages and showing that an individual laundry's contribution to total damages becomes negligible in the limit. To be able to show graphically the ways in which the influence of additional factories and laundries evolves as their numbers increase, we assume that (1) all factories are of the same size, and (2) the addition of a new factory moves them to a different "world" in which the size of every factory is smaller than before. Changing the number of factories therefore leaves total efficient output and total efficient emissions unchanged, while each factory's output and, hence, its share of total emission becomes smaller as the number of factories increases. In the limit, any single factory's contribution of emissions becomes negligible.

We adopt the same framework for laundries, assuming that (1) all laundries are of equal size, and (2) as the number of laundries increases, total laundry efficient output and, hence, total efficient damages remain unchanged, while each laundry's share of total damages falls. Thus, the addition of another laundry moves them to a different "world" in which each laundry produces less and incurs smaller damages from emissions, but because an additional laundry has entered, the aggregate marginal damage curve remains unchanged.

Our expositional "trick" of switching among different worlds, with factories and laundries of different sizes in each world while the numbers of factories and laundries change, enables us to show graphically what happens with large numbers of factories and laundries. However, the intuition behind our key result that the tax on laundries falls to zero as the number of laundries becomes large is that, in the limit, the reduction in efficient emissions caused by the marginal laundry increases the control costs of factories by exactly the same amount by which it reduces the damages to the other laundries. Hence, what matters is that an individual laundry's net contribution to damages becomes indefinitely small, and while switching to a world in which each laundry's output becomes smaller as the number of laundries increases achieves this result, it is not the only way of achieving it.

2.1 The change in the tax on factories as the number of factories increases

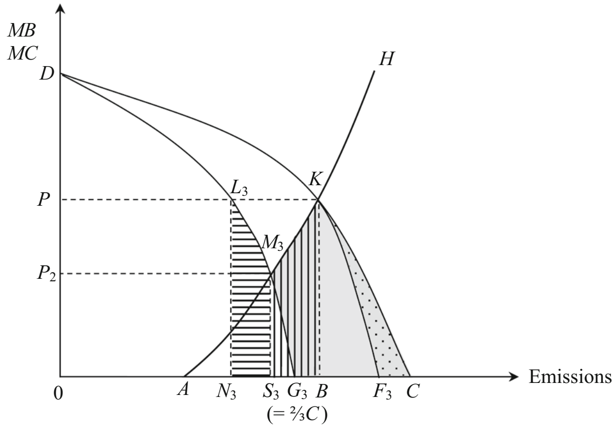
Figure 2 shows the situation with two factories and one laundry. Our assumptions imply that this move from a one-factory world to a two-factory world leaves the total amount of efficient emissions unchanged at *B* and the marginal cost of damages unchanged at *P*.

What would happen in this two-factory world if only one factory were to produce? The curve *DLG* measures the marginal benefit of emissions for each individual factory (which is half the size of the original factory in the single-factory world), while the curve *DKC* now measures the aggregate (horizontally summed) marginal-benefit-of-emission schedules of the two factories.[4] Point *G* is halfway between points 0 and *C*, and point *L* is halfway between points *P* and *K*. Each half-size factory would emit *S* if it was in its current size and the other factory did not operate, with marginal cost of damages of P_1. Thus, the production of the other factory increases efficient emissions from *S* to *B*, thereby leading to total emission costs of *AKC* rather than *AMG*, with an increase in emission costs of the grey area *GMKC*.

How much of this increase in the costs of total emissions are costs that the second factory imposes on the laundry and on the first factory, and how much are the second factory's control cost? First, the increase in emissions raises the marginal cost that the

[4] Thus, curve *DKC* measures the marginal benefit of the single factory's emissions in a one-factory world and the joint marginal benefit of the emissions of two factories in the two-factory world.

Fig. 3 The marginal cost of adding a third factory when all factories are of equal size



laundry bears, which is represented by *SMKB* (vertical stripes) underneath the marginal damages curve. Second, the increase in the marginal cost of emissions raises the cost of the marginal unit of emission from P_1 to P, which increases the control cost of the first factory by *NLMS* (horizontal stripes) underneath the first factory's marginal benefit curve *DLG*. Efficiency requires that the second factory bear these separate marginal costs that its existence imposes on the first factory and on the laundry. We show in the appendix that the sum of the areas *SMKB* and *NLMS* (= *NLMKB*) equals *GMKF* by construction. The remaining part of *GMKC*, the area formed by the dotted shape *FKC* underneath the new marginal benefit curve, *DKC*, is therefore the control cost that the second factory bears. Since each factory can be regarded as the marginal factory, each factory pays a tax equal to *NLMKB* (= *GMKF*), which is the combination of the marginal costs that the factory's efficient emissions impose on the other factory and the laundry, and it bears its own control cost *FKC*, hence bearing the full marginal cost *GMKC* that is caused by its emissions.

The standard Pigouvian approach is to charge each factory, on all its emissions, a tax equal to the damages of the marginal unit *B*. In terms of Fig. 2, the first factory's tax bill would be *P* times 0*N* (the rectangle 0*PLN*), and the second factory's total Pigouvian tax bill would be *P* times *NB* (the rectangle *NLKB*). However, as we show below, incentives for efficient emissions require that emitters bear not only the marginal cost of their marginal emissions, but also the actual incremental costs of every increment of their emissions. Thus, in a situation where individual emitters affect the price of emissions, efficiency in incentives is maintained if each factory bears the integral of the marginal costs of all its emissions, rather than the product of the cost of the marginal unit and the quantity of its emissions.[5] Nevertheless, the sum of the taxes that the two factories pay (twice the area *NLMKB*) exceeds the total damages *AKB* that their emissions impose on the laundry.

Figure 3 shows a world with three factories of equal size. The curve DL_3G_3 measures the joint marginal benefits of emitting for two factories, which would emit S_3 if they kept their current size and the third factory did not operate. The emissions of the third factory thus raise the marginal cost of emissions from P_2 to P, leading to efficient emissions of

[5] Consider a factory whose production process exhibits variable economies of scale, and whose marginal benefit of emissions increases initially and falls after crossing a threshold. While the integral of the marginal cost of emissions is less than the factory's total benefit from emitting, the product of the cost of the marginal unit and the quantity of its emissions might exceed this total benefit. In such a case, the standard Pigouvian tax leads to an inefficient location decision.

B and total emission cost of AKC, rather than AM_3G. The additional cost of G_3M_3KC is divided in the same way as in Fig. 2. Because all factories are of identical size, each factory pays a tax of N_3 L_3M_3KB (= $G_3M_3KF_3$) and bears its own control cost F_3KC.

Comparison of Figs. 2 and 3 shows that the addition of another factory reduces the area $NLMKB$ and raises the height of M. As the number of factories approaches infinity and the contribution of any individual factory towards total emissions becomes infinitesimally small, the shape $NLMKB$ approaches an infinitesimally thin rectangle of length KB. Thus, in the limit, each unit emitted becomes a marginal unit, so that each factory pays the amount P for each unit emitted, and the total tax collected equals the product of $0P$ and $0B$, which is the standard Pigouvian tax.

2.2 The change in the tax on laundries as the number of laundries increases

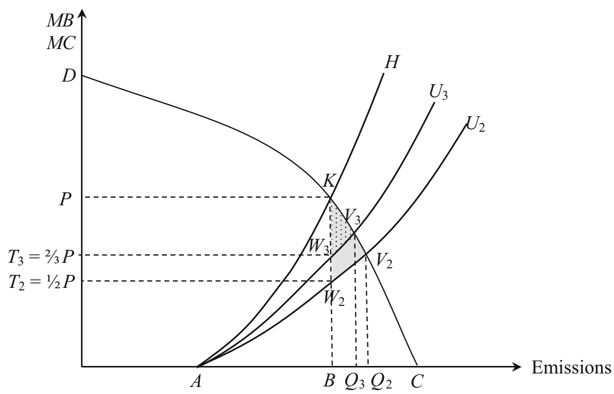
Figure 4 shows the derivation of the tax paid by laundries. The curve AH describes the marginal damages if only a single laundry operates. The efficient level of emissions is B, so the laundry pays a tax equal to the control cost BKC that factories bear because of the laundry's existence, and the laundry bears the damages from emissions, AKB.

In a world with two laundries, each laundry's size is half that of the laundry in a single-laundry world, and the marginal damages curve of each half-size laundry AU_2 has half the height and half the slope of the marginal damages curve of the single laundry. (As AU_2 crosses the line KB, its height T_2 is half the height P of the single laundry's marginal damages curve AH.) Because the vertical sum of two marginal damages curves AU_2 yields the marginal damages curve AH, total efficient emissions remain at B.

If only a single half-size laundry were to incur damages from emissions, then efficient emissions would be Q_2. Efficient emissions are B because the second laundry also incurs damages from emissions. Thus, the presence of the second laundry imposes control costs of BKV_2Q_2 on factories while reducing the other laundry's damages by $BW_2V_2Q_2$. The net cost of the second laundry's existence is therefore the grey area W_2KV_2, which each of the two half-size laundries pays as tax.

In a world with three laundries, the curve AU_3 measures the marginal damages to two laundries that have a combined size of two-thirds the size of the laundry in a single-laundry "world." Efficient emissions would be Q_3 if these two were the only laundries harmed by emissions. Efficient emissions are again B if the third laundry incurs damages from emissions. Thus, the presence of the third laundry imposes control costs of BKV_3Q_3 on

Fig. 4 The marginal cost of adding additional laundries when all laundries are of equal size



factories while reducing the damages that the other two laundries incur by the area $BW_3V_3Q_3$. The net cost of the third laundry's existence is therefore the dotted grey area W_3KV_3, which each of the three laundries pays as tax.

Comparison of the areas W_2KV_2 and W_3KV_3 indicates that the addition of a laundry reduces the net cost that is caused by each individual laundry's existence. As the number of laundries approaches infinity and the net cost caused by any individual laundry becomes infinitesimally small, the shape W_iKV_i shrinks towards point K, so that its area approaches zero. In the limit, the total tax paid by all laundries is zero.

2.3 The revelation of marginal costs and benefits

Vickrey (1961) showed that consistent application of marginal cost pricing can provide incentives to the parties involved to disclose honestly their marginal costs and benefits. Our Figs. 2 and 3 are simplified versions of Fig. 2 in Vickery (1961), and our tax schedules provide factories as well as laundries with the incentives to reveal their respective marginal cost and benefit schedules.

To explain why this is the case, we drop our expositional "trick", which assumes that the addition of factories and laundries keeps total emissions and damages unchanged, and re-interpret Figs. 2 and 4 in the conventional way, when the addition of another factory shifts the total marginal benefits curve outward while the addition of another laundry shifts the total marginal cost curve inward. Thus, the marginal benefit curve *DKC* shifts upward and outward when the second factory overstates its marginal benefits, and it shifts downward and inward when it understates them. Similarly, the marginal cost curve *AH* shifts upward and inward when the second laundry overstates its marginal costs, while it shifts downward and outward when it understates them.

Consider the presence of the second factory in Fig. 2, which shifts the marginal benefit schedule from *DMG* to *DKC* so that the second factory pays a tax equal to *NLMKB*. If the second factory overstates its marginal benefit *P* from emitting by enough to increase the apparently efficient emissions by one unit, then the marginal benefit schedule *DKC* shifts up. Thus, the area *NLMKB* increases by more than *P*, and the factory pays more than the marginal benefit for the marginal unit of emission. Similarly, if the second factory understates its marginal benefit from emitting by enough to reduce the apparently efficient emissions by one unit, then the marginal benefit schedule *DKC* falls, the area *NLMKB* decreases by more than *P*, and the factory forgoes the opportunity to emit at a cost that is below the emission's marginal benefit. Since the marginal cost of any misrepresentation of its marginal damages exceeds its marginal benefit of doing so, the second factory does best by announcing its true marginal damages.

A corresponding argument applies to laundries. Consider the situation of the two-laundry world in Fig. 4: if the second laundry overstates its marginal damages, then the marginal cost schedule *AH* rises, thereby increasing the apparent net marginal damages of the second laundry beyond W_2KV_2. Because they bear the cost of higher damages at the margin, factories reduce their emissions below *B*, thereby increasing their control costs and, thus, the tax that the second laundry must pay. Overstating its marginal damages therefore increases the second laundry's tax at the margin by an amount larger than the distance KW_2, while the marginal benefit that the second laundry receives from the additional reduction in emissions is smaller than KW_2. Similarly, if the second laundry understates its marginal damages so that the marginal cost schedule *AH* falls, then factories have an incentive to emit more than *B*, thereby lowering the second laundry's taxes at the margin by an amount smaller than the distance KW_2, while the marginal cost that the second laundry bears because of the additional emissions exceeds KW_2. Since the marginal cost of any misrepresentation of its marginal damages exceeds its marginal benefit of doing so, the second laundry does best by announcing its true marginal damages.

Vickrey applied his mechanism to the production and sale of a rival good, which leads to a budget deficit since buyers pay the marginal cost of each unit they consume while sellers receive, for each unit they sell, the marginal benefit that the buyer reports. Since marginal benefit equals marginal cost only for the marginal unit, each infra-marginal unit contributes to the deficit. In contrast, our application of Vickrey's construct to a non-rival bad is akin to the familiar VCG mechanism, which requires that each agent pay a tax equal to the net cost of her participation in a decision about a public good and usually leads to a budget surplus.[6]

While the continuous VCG mechanism is commonly applied to the provision of non-rival goods, our discussion indicates that it can be adapted easily to the regulation of non-rival bads.[7] In applications to non-rival goods, any (pivotal) agent whose expressed demand for the non-rival good changes the chosen quantity of the non-rival good imposes an indirect cost on the other agents, who either contribute more to the non-rival good or receive smaller quantities than they would have received otherwise. In applications to non-rival bads, any agent whose disclosed cost lowers the efficient quantity of the bad imposes a cost in form of a higher Pigouvian tax on the agents who cause the non-rival bad, in addition to providing a benefit to those agents who now suffer from a smaller quantity of the non-rival bad.

Figure 4 indicates that a laundry's tax is represented by a triangle that reduces to a dot of measure zero when the laundry claims only marginal damages. Such a tax increases (approximately) quadratically with the damages that the laundry claims. Such quadratic taxes are a general characteristic of demand-revelation mechanisms for non-rival goods, including the VCG mechanism and the types of mechanisms represented by quadratic voting, the Groves–Ledyard mechanism and the Hylland–Zeckhauser mechanism. Hence, our analysis highlights the fact that, in these mechanisms, the payments for altering the social choice become insignificantly small as the number of agents who cause such indirect costs becomes large and the change in the efficient quantity that each individual agent requests becomes negligible. In contrast, actions that create non-excludable bads—like the emissions of factories—lead to payments that do not disappear when the number of agents who cause such direct costs becomes large, because the harm of the marginal unit does not disappear. Hence in the limit, these payments become linear as every agent pays the damages of the marginal unit of the bad, and the total payment is the product of the marginal damages and the number of units of emissions.[8]

6 We write "usually" because while the VCG mechanism requires that each agent bear the net cost of her presence, it does not require that she pay this amount. The VCG mechanism also can be implemented by offering each agent a subsidy, equal to the net social benefit from her not affecting the social choice as much as she might have. By revealing her true benefit schedule, the agent ensures that her net marginal benefit on the marginal unit of change that is caused by her presence equals the foregone subsidy that she would have received had she not caused this marginal unit of change. Financing the subsidies would then lead to a budget deficit as well.

7 Kunreuther and Kleindorfer (1986, p. 295) claim that VCG mechanisms cannot be applied to the regulation of public bads since such an application "requires the public commodity to have a positive value to each participant so that there is a net surplus after the commodity tax is levied." Our Fig. 2 shows that the area 0DKMLA represents such a net surplus even after all taxes are paid. Kunreuther and Kleindorfer reference Tideman and Tullock (1976) in support of their claim, but we are not aware of any statement in Tideman and Tullock that would provide such support.

8 We thank Glen Weyl for pointing this characteristic of VCG mechanisms out to us.

3 Convex opportunities—mathematical argument

Consider a region with n identical factories and m identical laundries. The combined output of all factories is X. The production of total factory output X leads to total emissions E (point B in Fig. 3) that harm the operation of laundries at an increasing rate. The combined output of all laundries is $Y(E)$. Because of the assumption of identical factories and identical laundries and the assumption that X and $Y(E)$ do not change with the numbers of factories and laundries, the efficient emissions of factory j are $e_j^* = E/n$, the efficient output of factory j is $x_j^* = X/n$, and the efficient output of laundry i is $y_i^*(E) = Y(E)/m$. These assumptions ensure that each factory's efficient output and emissions as well as each laundry's efficient output becomes inconsequentially small relative to the respective total efficient output as well as to total efficient emissions, as the numbers of factories and laundries approach infinity.

3.1 The tax on factories

Let $\Pi_x(E/n)$ denote each factory's profit and let $\Pi_y(Y(E)/m)$ denote each laundry's profit when all factories and laundries produce their respective efficient levels of output. We assume that emissions harm the operation of laundries at an increasing rate, so $\frac{\partial \Pi_y(Y(E)/m)}{\partial E} < 0$ and $\frac{\partial^2 \Pi_y(Y(E)/m)}{\partial E^2} < 0$. The government imposes, on the emissions e_j of each factory j, a Pigouvian tax equal to the sum of the costs to other factories of cutting back their emissions and the costs to laundries of enduring the net increase in emissions.

If factory j emits $e_j^* = \frac{E}{n}$, then the other $n - 1$ factories emit $E - e_j^* = E\frac{(n-1)}{n}$ (point N_3 in Fig. 3). If factory j were to stop production and hence reduce its emissions to zero, then $\frac{\partial^2 \Pi_y(Y(E)/m)}{\partial E^2} < 0$ implies that the marginal cost that emissions impose on laundries and, hence, the Pigouvian tax on the marginal unit of emission would fall, and the other $n - 1$ factories would increase their emissions to E_{-j} (point S_3 in Fig. 3). Thus, factory j's efficient emissions e_j^* lower the emissions of the other $n - 1$ factories by $E_{-j} - (E - e_j^*)$ (the distance between points S_3 and N_3 in Fig. 3) and increase net emissions by $E - E_{-j}$ (the distance between points B and S_3 in Fig. 3).

Let $T_j^1(e_j^*)$ denote the total costs that factory j's efficient emissions impose on laundries, and let $T_j^2(e_j^*)$ denote the total costs to the other $n - 1$ factories that arise from constraining their emissions in response to factory j's efficient emissions. If the government requires factory j to bear the full cost of its emissions, then the two taxes that factory j pays for its effects on laundries and other factories respectively, when it emits e_j^* are

$$T_j^1\left(e_j^*\right) = \int_{E_{-j}}^{E} m\frac{\partial \Pi_y(Y(e)/m)}{\partial e}de \tag{1}$$

and

$$T_j^2\left(e_j^*\right) = \int_{E-e_j^*}^{E_{-j}} (n-1)\frac{\partial \Pi_x(e/n)}{\partial e}de. \tag{2}$$

Note that the integrands of Eqs. 1 and 2 have the same values at the opposite integration limits: First, when factory j emits e_j^*, the other $n - 1$ factories emit $E - e_j^* = E\frac{(n-1)}{n}$. Hence, the joint marginal value of an additional unit of emissions when the aggregate emissions of the $n-1$ factories are $E - e_j^*$ must equal the cost of the marginal unit of emission at E, or

$$(n-1)\frac{\partial \Pi_x(e/n)}{\partial e}\bigg|_{e=E-e_j^*} = m\frac{\partial \Pi_y(Y(e)/m)}{\partial e}\bigg|_{e=E^*}. \tag{3}$$

In terms of Fig. 3, the marginal value of additional emissions to the first two factories when their aggregate emissions are N_3 must equal the marginal cost of additional emissions at B, and both must equal P.

Second, if factory j were to stop production and reduce its emissions to zero, then the cost of the marginal unit of emission would fall and the other factories would increase their emissions until their joint marginal value of additional emissions at E_{-j} equaled the cost of the marginal unit of emission at E_{-j}, or

$$(n-1)\frac{\partial \Pi_x(e/n)}{\partial e}\bigg|_{e=E_{-j}} = m\frac{\partial \Pi_y(Y(e)/m)}{\partial e}\bigg|_{e=E_{-j}}. \tag{4}$$

In terms of Fig. 3, when the third factory does not emit, the marginal benefits of emissions for the first two factories must equal the marginal cost of additional emissions at point S_3.

How would factory j's total tax change if factory j's emissions were to increase by a marginal amount? The derivatives of Eqs. 1 and 2 with respect to e_j^* are

$$\frac{dT_j^1\left(e_j^*\right)}{de_j^*} = -m\frac{\partial \Pi_y(Y(e)/m)}{\partial e}\bigg|_{e=E_{-j}} \tag{5}$$

and[9]

$$\frac{dT_j^2\left(e_j^*\right)}{de_j^*} = \int_{E-e_j^*}^{E_{-j}} (n-1)\frac{d\left(\frac{\partial \Pi_x(e/n)}{\partial e}\right)}{de_j^*}de + (n-1)\frac{\partial \Pi_x(e/n)}{\partial e}\bigg|_{e=E_{-j}} + (n-1)\frac{\partial \Pi_x(e/n)}{\partial e}\bigg|_{e=E-e_j^*}. \tag{6}$$

Adding Eqs. 5 and 6 and using Eqs. 3 and 4 yields

$$\frac{dT_j^1\left(e_j^*\right)}{de_j^*} + \frac{dT_j^2\left(e_j^*\right)}{de_j^*} = \int_{E-e_j^*}^{E_{-j}} (n-1)\frac{d\left(\frac{\partial \Pi_x(e/n)}{\partial e}\right)}{de_j^*}de + m\frac{\partial \Pi_y(Y(e)/m)}{\partial e}\bigg|_{e=E-e_j^*}. \tag{7}$$

The integral in Eq. 7 is zero when factory j exists but produces nothing, because then $e_j^* = 0$ and $E_{-j} = E - e_j^* = E$. Thus, the marginal tax when factory j emits marginally more than zero is

[9] The existence of the integral in Eq. 6 is a consequence of our assumption that total emissions are constant. If the emissions of one factory increase, then all other factories must emit less—that is, when an additional factory incurs marginal damages, we move to a world in which the marginal benefit from emitting is smaller than it was before because all factories have shrunk in size.

$$\frac{dT_j^1\left(e_j^*\right)}{de_j^*}+\frac{dT_j^2\left(e_j^*\right)}{de_j^*}=m\frac{\partial \Pi_y(Y(E)/m)}{\partial E}. \tag{8}$$

In the limit, when every factory emits an inconsequentially small share of total emissions, factory j's efficient emissions e_j^* equal the marginal unit of emissions, so that $e_j^* = de_j^*$. Thus, in the limit, each factory pays the marginal damages of emitting the marginal unit on all units emitted, and the sum of all taxes on factories equals $B\,\frac{\partial \Pi_y(Y(E)/m)}{\partial E}$.

3.2 The tax on laundries

Let E denote total factory emissions if laundry i produces y_i^* and let E_{-i} denote total emissions if laundry i exists but does not operate, where $E_{-i} > E$ because laundry i's operation increases each factory j's tax $T_j^1(e_j^*)$ and, hence, its marginal cost of operating. Thus, the cost that factory emissions impose on laundry i's operation when efficiency is attained, and that the n factories bear, is

$$A_1=\int_E^{E_{-i}} n\frac{\partial \Pi_x(e/n)}{\partial e}de, \tag{9}$$

which is the area of BKV_3Q_3 in Fig. 4, for $m = 3$. The reduction in emissions from E_{-i} to E lowers the damages on the other $m - 1$ laundries and, hence, increases their profit by

$$A_2=\int_E^{E_{-i}} (m-1)\frac{\partial \Pi_y(Y(e)/m)}{\partial e}de, \tag{10}$$

which is the area of $BW_3V_3Q_3$ in Fig. 4, for $m = 3$. If the government requires that laundry i bear the net cost that its operation imposes on the other agents, then laundry i pays a tax equal to $T_i(y_i^*) = A_1 - A_2$.

The two integrands in Eqs. 9 and 10 have the same value at the upper limit of integration, where emissions are E_{-i} (laundry i exists but its output is zero),

$$n\frac{\partial \Pi_x(e/n)}{\partial e}\bigg|_{e=E_{-i}}=(m-1)\frac{\partial \Pi_y(Y(e)/m)}{\partial e}\bigg|_{e=E_{-i}}, \tag{11}$$

because the Pigouvian tax on the n factories provides them with the incentive to generate emissions up to the level at which the marginal benefit of the last unit of emission equals the marginal cost. A marginal increase in laundry i's output increases the laundry's tax according to

$$\frac{dT_i\left(y_i^*\right)}{dy_i^*}=n\frac{\partial \Pi_x(e/n)}{\partial e}\bigg|_{e=E_{-i}}-(m-1)\frac{\partial \Pi_y(Y(e)/m)}{\partial e}\bigg|_{e=E_{-i}}-\int_E^{E_{-i}}(m-1)\frac{d\left(\frac{\partial \Pi_y(Y(e)/m)}{\partial e}\right)}{dy_i^*}de, \tag{12}$$

which, using Eq. 11, simplifies to

$$\frac{dT_i\left(y_i^*\right)}{dy_i^*}=-\int_E^{E_{-i}}(m-1)\frac{d\left(\frac{\partial \Pi_y(Y(e)/m)}{\partial e}\right)}{dy_i^*}de. \tag{13}$$

The integrals in Eqs. 12 and 13 reflect our assumption that total emissions and, hence, the marginal damages from emissions are constant. If one laundry's marginal damages increase, then the marginal damages of the other laundries must decrease—that is, when an additional laundry incurs marginal damages, we move to a world in which each laundry's marginal damages from emissions are smaller than before because all laundries shrink in size. The derivative within the integral indicates how additional laundry output affects the reduction in the laundry's profit from the marginal harm of emissions; this effect is negative because the marginal damages of emissions are larger when the laundry produces more. Because the tax on factories increases to accommodate this increase in damages, laundry i's tax increases when it produces more.

The integral in Eq. 13 and, hence, the laundry's marginal tax is zero when laundry i exists but does not produce anything, because $E_{-i} = E$ in this case. Thus, when each laundry's share of total laundry output becomes inconsequentially small, no laundry pays a tax, and the sum of the taxes on the indefinitely many laundries is zero.

3.3 A comparison of the taxes on factories and laundries

A comparison of the marginal taxes described by Eqs. 7 and 13 provides additional insights. In both equations, the integrals represent the respective net increases in the factories' control costs that are caused by the higher tax on the marginal unit of emissions. A laundry that contributes only the marginal unit towards total damages raises the factories' control costs by exactly the amount by which it raises the other laundries' benefits from marginally cleaner air, so that the *net* increase in social costs is zero. Thus, the integral in Eq. 13 disappears in the limit. Similarly, a factory that contributes only the marginal unit towards total emissions does not raise the control costs of the other factories, because the marginal unit of emissions is already priced at the marginal damage that it causes. Thus, the integral in Eq. 7 disappears in the limit. Both instances are examples of the familiar result in public finance that "the first marginal distortion is free," because the cost of the first marginal distortion is exactly offset by its benefits.[10]

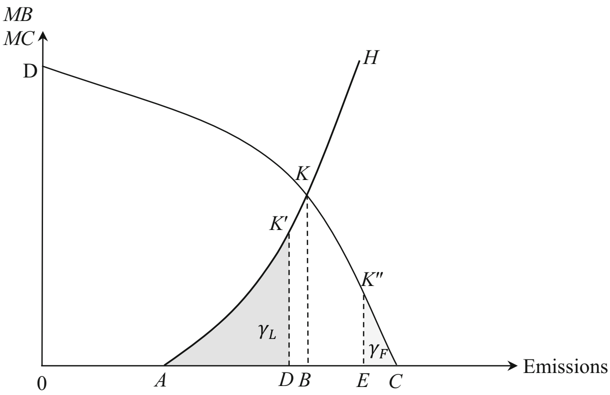
The second term in Eq. 7 represents the damage on the m laundries that is caused by the marginal unit of emissions, and even a factory that produces an inconsequentially small amount of total output pays the damage that is caused by its (marginal unit of) emissions. If every factory generates only a marginal unit of emissions, then the total Pigouvian tax that is paid by all factories is the product of the total number of units emitted and the costs per unit. Thus, in the limit, the costs of factories follow a linear pricing model, where all factories pay the same price for the marginal units that they emit, while the quadratic net cost of each laundry approaches zero.

4 Non-convexities

Thus far, we have assumed the existence of an interior efficient solution, at which laundries incur some damages from emissions and factories emit less than they would in the absence of laundries. Consider such an interior efficient solution, in which n factories and m laundries operate, total damages incurred by laundries are $\Gamma_L = AKB = \int_A^B m \frac{\partial \Pi_y(Y(e)/m)}{\partial e} de$,

[10] See, for example, Tresch (2002, p. 417).

Fig. 5 Non-convexities



total factory control costs are $\Gamma_F = KBC = \int_B^C n \frac{\partial \Pi_x(e/n)}{\partial e} de$, and the full cost is $\Gamma = AKC = \Gamma_L + \Gamma_F$.

Now introduce the possibility of either party resolving the conflicting situation on its own, albeit at a cost. Let γ_F be the total control cost to factories of eliminating all harm to laundries, either by relocating or by abating emissions to A or less through a joint scrubbing facility or some other possibility that introduces a non-convexity into control costs. Similarly, let γ_L be the total cost to laundries of preventing any damages, either by relocating or installing a joint air cleaning facility or other device that ensures that emissions cause no damage. In Fig. 5, the cost to factories γ_F is the area $EK''C$ and the cost to laundries γ_L is the area $AK'D$.

If $\gamma_F > \Gamma$ and $\gamma_L > \Gamma$, then the interior solution is optimal. If $\gamma_F < \Gamma$ and/or $\gamma_L < \Gamma$ and if bargaining between the two sides is not trusted to solve the problem, then efficiency requires that the party with the smaller cost act, and to motivate that action each party must be required to pay at least $min\{\gamma_F, \gamma_L\}$. Thus if $\gamma_F < \Gamma$ and $\gamma_F < \gamma_L$, as in Fig. 5, then efficiency requires that factories reduce emissions to A or less. Requiring factories to pay for all harm caused by their emissions motivates them to do so, provided that they can solve the associated coordination problem, if coordination is required. Laundries have an incentive to reduce harm efficiently if they bear the full cost caused by their presence. Laundries whose presence causes control costs that they are not required to pay might locate near factories even if their marginal cost from locating elsewhere is smaller than the factories' control costs. Similarly, laundries that can install a joint air cleaning facility at a cost below the factories' control costs have no incentive to do so if they do not bear these control costs. Thus, in this case of $\gamma_F < \Gamma$ and $\gamma_F < \gamma_L$, efficiency requires that laundries pay a tax γ_F so that each party bears the full cost γ_F.

If $\gamma_L < \Gamma$ and $\gamma_L < \gamma_F$, then efficiency requires that laundries avoid all damages and factories emit at C. Requiring laundries to pay for all control costs that factories incur motivates laundries to do so. Factories that are allowed to ignore the costs of their emissions might locate next to the laundries even if the marginal benefit of relocation is less than γ_L, or they will continue to emit C even if $\gamma_F < \gamma_L < \Gamma_L$, and then rely on the laundries to resolve the conflict by spending γ_L. Again, efficiency requires that factories pay a tax γ_L so that each party bears the full cost γ_L.[11]

Our proposed accommodation of non-convexities works well as long as only a single factory and a single laundry are involved. We acknowledge that the collective action problems associated with requiring multiple factories and laundries to bear the respective costs of γ_L and γ_F are likely to render our proposal ineffective in practice. As an example, consider the collective action problem of laundries when $\gamma_L < \gamma_F < \Gamma$. Laundries must decide whether to identify themselves as victims and pay γ_L to avoid all damages from emissions, or relocate at a total cost of $\rho_L = \sum_{i=1}^{m} \rho_{Li}$, where ρ_{Li} is the relocation cost of laundry i. Because the joint air cleaning facility is a non-rival good, laundries can, in theory, use the VCG mechanism to determine each laundry's willingness to contribute to γ_L. However, for the VCG mechanism to lead to efficient location decisions, it must be possible to identify and tax every laundry affected by emissions, including those laundries that are considering whether or not to move into the area, and enforce contributions from any laundries that have had their costs and benefits misestimated and would rather leave the area than pay their assigned contributions to γ_L.

To understand why this is the case, assume first that it is possible to identify and tax every laundry that might be affected by emissions and that, if installing the air cleaning facility is efficient, laundries must contribute towards γ_L and can neither relocate nor close. The VCG mechanism is then implemented by assigning, to each affected laundry i, a contribution γ_{Li}^e that is based on an estimate of the laundry's relative damages, with $\gamma_L = \sum_{i=1}^{m} \gamma_{Li}^e$. Each laundry i announces its net benefit ω_{Li} of paying γ_{Li}^e and having the air cleaning facility installed, where laundries that would rather relocate than pay γ_{Li}^e announce $\omega_{Li} < 0$. A negative announced total net benefit, $\sum_{i=1}^{m} \omega_{Li} < 0$, implies $\rho_L < \gamma_L$, indicating that it is efficient not to install the air cleaning facility and for all laundries to relocate. A positive announced total net benefit, $\sum_{i=1}^{m} \omega_{Li} > 0$, implies $\gamma_L < \rho_L$, indicating that it is efficient to install the air cleaning facility.[12] Laundries that announce $\omega_{Li} < 0$ must pay γ_{Li}^e nevertheless.

If laundries that do not wish to pay γ_{Li}^e are allowed to relocate instead and some laundries exercise this option in order to escape their obligations to pay, then these relocations are inefficient. The case of $\sum_{i=1}^{m} \omega_{Li} > 0$ is resolved without the possibility of such

[11] Consider Coase's original example of a factory whose emissions cause annual damages worth $\Gamma = \$100$ to a nearby resident. The resident can relocate at an annual cost of $\gamma_L = \$40$, while the factory can abate all emissions at an annual cost of $\gamma_F = \$90$. Because Coase (1960, p. 41) assumed that the factory is taxed \$100 per year if it emits, taxing the factory but not the resident leads to socially inefficient spending of \$90 – \$40 = \$50. Baumol (1972, p. 472, fn. 1) argued that the true marginal damage and, hence, the appropriate Pigouvian tax is zero rather than \$100, because it is socially optimal for the resident to move, which he will do on his own in the absence of the factory tax. However, Thompson and Batchelder (1974, p. 470) pointed out that the appropriate Pigouvian tax on the factory is $\gamma_L = \$40$, so that each party bears the full social cost γ_L.

[12] To provide laundries with an incentive to reveal their respective net benefits, each laundry i whose announcement ω_{Li} causes the sign of $\sum_{j=1}^{m} \omega_{Lj}$ to differ from the sign of $\sum_{j=1, j \neq i}^{m} \omega_{Lj}$ (a "pivotal" laundry) must pay a Clarke tax equal to the absolute value of $\sum_{j=1, j \neq i}^{m} \omega_{Lj}$. For example, if $\sum_{j=1, j \neq i}^{m} \omega_{Lj} < 0$ and $\sum_{j=1}^{m} \omega_{Lj} > 0$, then the joint air cleaning facility would not have been installed had laundry i announced a net benefit less than $\left| \sum_{j=1, j \neq i}^{m} \omega_{Lj} \right|$, but the facility is installed because laundry i has announced $\omega_{Li} > \left| \sum_{j=1, j \neq i}^{m} \omega_{Lj} \right|$. Laundry i therefore pays a Clarke tax equal to $\left| \sum_{j=1, j \neq i}^{m} \omega_{Lj} \right|$, which is the margin by which those in favor of rejecting the facility would have won in laundry i's absence. Thus, each laundry pays the cost that its announcement imposes on all other laundries. Plassmann and Tideman (2011) offer a related application of the VCG mechanism to the problem of land assembly.

inefficiency only if either taxes cannot be escaped by relocation or $\gamma_{Li}^e < \rho_{Li} \forall i$, so that every laundry announces a positive ω_{Li}, indicating that it prefers to pay γ_{Li}^e rather than relocate. The case of $\sum_{i=1}^m \omega_{Li} > 0$ is not resolved efficiently if $\gamma_{Li}^e > \rho_{Li}$ for some $k < m$ laundries that will relocate rather than pay γ_{Li}^e, because $\sum_{i=1}^m \omega_{Li} > 0$ implies $\gamma_L < \rho_L$, indicating that there exists an alternate set of contributions for which every laundry's contribution is less than its cost of relocation. Thus, it would be optimal to install the air cleaning facility and for all laundries to remain in the area.[13]

A similar problem arises if it is not possible to identify and include in the VCG process every laundry that might be affected by emissions. Assume that $\sum_{i=1}^m \omega_{Li} < 0$ for the m laundries currently located within the area, while $\sum_{i=1}^q \omega_{Li} > 0$ for the $q > m$ laundries that include those that are still considering whether or not to locate there. The inability to include all q laundries in the decision-making process prevents the efficient installation of the air cleaning facility.

Given these difficulties, it is generally impractical to accommodate non-convexities by relying on the parties involved to resolve all collective action problems on their own, and it is more appropriate to rely on the coercive power of the state to implement general rules that are supported by the citizenry at large. For example, new residents who move to rural communities often complain about farming noises and smells. In response to such complaints, all states in the United States have adopted right-to-farm laws that deny nuisance law suits against commercial farmers, thereby establishing the farmers' right to engage in established farming activities. Thus, even though farmers might sometimes be able to adapt their farming practices at costs below those borne by the new residents, the social cost of categorically denying these types of nuisance law suits is almost certainly below those of individually resolving each such complaint efficiently.

5 Bilateral taxes and fees in the economics literature

Vickrey (1961) introduced the idea of charging all parties to an interaction the marginal cost of their respective actions, and Coase (1960) applied this idea to the economics of pollution.[14] Later, Zeckhauser (1968) and Vickrey (1968) introduced the idea of consistent marginal cost pricing in the theory of accidents. Theodore Groves, in his 1970 doctoral dissertation, applied a reverse version of the same principle to the design of efficient incentives in teams—team members have incentives to behave efficiently if every team member is paid the entire benefit of the team's joint output.[15] In the 1970s, Clarke (1971, 1972), Groves and Loeb (1975), Groves and Ledyard (1977), Arrow (1979), D'Aspremont and Gérard-Varet (1979) and Hylland and Zeckhauser (1979) sparked a new line of research by using marginal cost pricing to provide incentives for the truthful

[13] Applying the VCG mechanism again among the remaining $m - k$ laundries leads to one of two possible scenarios: in scenario 1, the air cleaning facility is installed because $\omega_{Li} > 0 \ \forall i = 1, \ldots, m - k$ (possibly after several iterations during which additional laundries leave), although the laundries that relocated could have remained in the area at no cost. In scenario 2, the air cleaning facility is not installed because $\sum_{i=1}^{m-k} \omega_{Li} < 0$, and all laundries leave the area, even though there was a different set of γ_{Li}^e's $\forall i = 1, \ldots, m$ under which all laundries would have preferred to stay.

[14] Even though the date on Coase's publication precedes that of Vickrey's paper, Vickrey might deserve credit for an earlier publication because the 1960 issue of the *Journal of Law and Economics* has a 1961 copyright date. The issue arrived at the library of the University of Virginia in April 1961.

[15] This approach might be financially feasible if team members must pay fixed fees to be on a team and a person not on the team collects the fees and pays the rewards.

revelation of preferences for public goods. Recent work in quadratic voting by Weyl (2012), Goeree and Zhang (2013) and Lalley and Weyl (2016) continues to advance that tradition.

The idea of bilateral Pigouvian taxes introduced in Coase (1960) produced a flurry of responses. Buchanan and Stubblebine (1962) showed that unilateral Pigouvian taxes generally do not equalize marginal cost and marginal benefit, which is correct except in the limit when the number of recipients of harm becomes indefinitely large. Baumol (1972, p. 307) suggested that, in the "large numbers case," the taxation of laundries is incompatible with efficient allocations.[16] Thompson and Batchelder (1974) pointed out that Baumol did not specify whether he was referring to large numbers of factories or large numbers of laundries, and they showed, correctly, that if a single laundry operates, efficiency requires a tax on the laundry. Baumol acknowledged their argument in Baumol (1974) and Baumol and Oats (1975).

Mohring and Boyd (1971) discussed a situation of waste discharge into a river, where initial disposal causes no damages. An entrepreneur considers setting up a bathing beach, at a location either upstream or downstream of the waste-disposing factory. Mohring and Boyd showed that if the factory is required to pay for any damages that it causes, the entrepreneur can make an efficient location decision only if he accounts for the control cost that the factory will incur when the beach is established downstream of the factory. Hence, as with the laundries in our example, efficiency requires that the entrepreneur bear the marginal control cost that his presence imposes on the factory.

Baumol (1972) considered the regulator's problem of identifying marginal damages at the social optimum, to set the efficient Pigouvian tax on those who cause the externality. He showed that an iterative sequential adjustment mechanism leads to the social optimum when this optimum is unique, but not when multiple local maxima exist. Kraus and Mohring (1975) concluded that when there are multiple local maxima, an iterative adjustment mechanism will not guarantee the socially optimal outcome under either unilateral or bilateral taxes. White and Wittman (1982) developed a spatial model in which factories and residents bid for land use and these bids provide enough information for landowners to identify globally optimal land users. They showed that in such a model with iterative adjustment of taxes, bilateral taxes are necessary for long-run efficient allocations.

Authors who advocate bilateral Pigouvian taxes do not always relate them to marginal cost pricing. For situations with a single laundry, Buchanan and Stubblebine (1962) as well as Ng (1971, 2007) specified the magnitude of the laundry tax in the same way that we do, that is, as the factory's control cost *KBC* in Fig. 1. Buchanan and Stubblebine (1962) did not specify whether and how they would modify the tax for multiple laundries, while Ng (2007) suggested dividing the total control cost when all laundries are present according to the relative differences in the laundries' damages, which he assumes to be known. Thus, Ng's tax on multiple laundries is a function of each laundry's damages, rather than of the marginal effects that each laundry has on (1) the factory's control cost and on (2) the reduction of the damages incurred by the other laundries. Because Ng's tax does not fully capture (2), it does not lead to efficient location decisions for large laundries that incur disproportionally large damages while their presence lowers the damages of the other laundries by large amounts.

Thompson and Batchelder (1974, p. 470) suggested that "laundry production near factories should be taxed to the extent that such production lowers their costs by inducing factories to create less smoke." This tax eliminates the external benefit that laundries as a

[16] See also Schulze and D'Arge (1974, p. 766, fn 8).

group receive from producing more, rather than charging laundries for the marginal cost their operations impose on the factory. Thompson and Batchelder did not specify how they would apportion the tax on laundry production among multiple laundries. Finally, White and Wittman (1982) considered a laundry tax that equals the factories' cost of abatement—hence, if factories pay their tax but spend nothing on abatement, then laundries pay no tax.[17]

With the exception of Ng (2007), the environmental economics literature has not examined bilateral Pigouvian taxation any further since the mid-1980s. Such a development would be appropriate if the relevance of bilateral taxation had become part of the pool of common knowledge in economics. But undergraduate as well as graduate textbooks that discuss Pigouvian taxes do not mention that efficiency requires taxing those affected by an externality, whenever their number is finite. Most textbooks in public economics and environmental economics discuss Pigouvian taxes in the context of situations with many harmed parties, making their exclusive focus on Pigouvian taxes on emitters defensible.[18] However, the 9th edition of the popular public finance textbook by Harvey Rosen and Ted Gayer introduces Pigouvian taxes in an example with two persons, Bart and Lisa; Rosen and Gayer mention only the effect of taxing Bart, who causes the externality, but ignore the effect of not taxing Lisa, who is affected by Bart's externality.[19] One might argue that ignoring bilateral Pigouvian taxation is acceptable when Pigouvian taxes are covered in just one or two pages. Yet such an omission misses an opportunity to teach students the complex role that marginal cost pricing plays in achieving efficiency.

6 Further applications of bilateral marginal cost pricing

Besides applications to negative externalities, accidents and mechanisms for the truthful revelation of preferences, bilateral marginal cost pricing can advance our understanding of various social interactions in which the parties impose costs on each other that are not reflected in market prices. We discuss the application of bilateral marginal cost pricing to two types of "attractive nuisances."

In the law of torts, an attractive nuisance is a situation that attracts a person—usually a child—who is unable to identify associated risks and who is put in danger by the existence of the attractive nuisance. For example, unguarded private swimming pools might attract children who disregard the risks of unsupervised swimming and who subsequently drown. In response, many localities in the United States have passed laws that require pool owners to surround their pools with fences or other obstacles to make it impossible for children to enter a pool in the owner's absence.

The requirement to erect a fence represents a Pigouvian tax on the pool's owner. Let γ_L be the cost of a lost life and γ_F be the cost of a fence around the pool. Since $\gamma_L > \gamma_F$,

[17] In their spatial model, laundries and factories compete with each other for locations. Hence, a laundry will choose to locate next to a factory that pays the tax rather than abate only if it is efficient to do so. White and Wittman did not address the concern raised by Buchanan and Stubblebine (1962) that laundries might try to extort payments from factories prior to divulging their final location decisions.

[18] See Mas-Colell et al. (1995, pp. 354–356), Tresch (2002, pp. 194–202, 212–228), Anderson (2003, pp. 103–108), Gruber (2007, pp. 134–146), Hyman (2008, pp. 104–110), Tresch (2008, pp. 106–119), Seidman (2009, pp. 36–43), Tietenberg and Lewis (2014, pp. 371–396), Callan and Thomas (2013, pp. 318–321) and Stiglitz and Rosengard (2015, pp. 139–140).

[19] Rosen and Gayer (2009, pp. 84–85). The example was already included in the 5th edition, published in 1999 by Harvey Rosen alone.

efficiency requires a fence that eliminates the possibility of a lost life. Because the fence is unnecessary if no children live nearby, efficiency requires that, if only a single family with children either lives nearby or considers moving into the neighborhood, that family bear the cost of the fence to be motivated to make an efficient location decision. However, as long as a fence that keeps one child away from the pool will keep all children away, the presence of a second family with children does not increase the cost to the pool's owner. Hence, when multiple families with children live close to the owner of a pool, efficiency requires that the pool owner bear the cost of the fence, while the families with children pay nothing.[20]

What is essential for the result that the taxes on laundries and families with children fall to zero as the numbers of laundries and families with children increases is that neither laundries nor families with children impose costs on the other party beyond control costs, and that the presence of any laundry and any child conveys benefits on other laundries and other families with children, respectively. Consider the related case of a steward of valuable items who does not guard them appropriately and thereby enables a thief to steal them. For the thief who gets caught and punished, such unguarded items represent an "attractive nuisance." Hence it is appropriate to discourage the steward from being negligent, by punishing him when his negligence is discovered, even if the negligence has not led to theft.[21] This threat of punishment is equivalent to the threat of punishing the owner of a pool who refuses to erect a fence, and the steward's cost of guarding the stores is equivalent to the cost of erecting the fence. However, the actions of the owners of laundries and children who want to swim impose only control costs on the owners of factories and pools, and they convey benefits to the other laundries when factories emit less and to other children when pool owners protect their pools, respectively. In contrast, the actions of thieves impose direct costs on the owners of the items with which the stewards are entrusted, and they do not convey comparable benefits to other thieves when stewards becomes more careful in guarding their stores. Thus, even as the number of thieves becomes large, it is nevertheless appropriate to punish all thieves.

7 Conclusion

We have resolved the disparate views of Coase (1960) and Baumol (1972) by showing that bilateral Pigouvian taxation provides incentives for efficient behavior as long as the number of those harmed by pollution is finite. In the limit, efficiency requires that only polluters be taxed.

Following the insights in Vickrey (1961), marginal cost pricing can be implemented in a way that provides polluters as well as pollutees with incentives to reveal honestly their respective marginal benefit and marginal cost schedules. Hence, bilateral taxation has the additional advantage of permitting a reduction in the uncertainty regarding the marginal benefit and marginal cost schedules that regulators need to know in order to levy the appropriate Pigouvian taxes. However, Pigouvian taxes entail additional administrative costs, including training and periodically auditing the regulators, identifying all parties

[20] Situations with multiple pool owners and multiple families with children lead to the collective action problem that we identified in Sect. 4. In such cases, it will generally be easiest to require every pool owner to secure her pool rather than resolve every individual situation efficiently.

[21] For example, the German army punishes soldiers who leave their lockers unsecured, thereby inviting their fellow soldiers to steal ("*Anleitung zum Kameradendiebstahl*").

involved, explaining and implementing the bilateral taxation scheme, as well as collecting the tax revenue. Because these administrative costs increase with the number of actors and because the tax on pollutees declines with the number of pollutees, it is probably not worthwhile levying taxes on pollutees when the tax on the marginal pollutee becomes sufficiently small. In addition, the possibility of reaching an efficient solution through bargaining suggests that taxation is probably not needed when only one polluter and one pollutee interact. Thus, in practice, bilateral Pigouvian taxes are likely to be warranted only when the number of pollutees is small, but larger than one.

Our limiting result that the tax on factories remains positive while the tax on laundries becomes inconsequentially small uses insights from the literature on demand revelation as well as from the model of linear market pricing. The literature on demand revelation has established that the efficient price for affecting the provision of non-rival goods is quadratic, which implies that this price is zero when one accepts the quantity chosen in one's absence and when the social cost of altering this quantity marginally is offset exactly by the benefit of doing so. Thus, a laundry whose presence leads to a marginal reduction in emissions pays no tax, because the higher tax on factories to create marginally cleaner non-rival air exactly offsets the benefit that the other laundries receive from such marginally cleaner air. In contrast, the model of linear market pricing for the harm of pollution has established that, in the limit, each supplier must bear the marginal social cost of producing the marginal unit. Thus, in the limit, all factories taken together must pay an amount equal the social cost of the marginal unit of emissions multiplied by the total number of units that they emit.

We have couched our analysis in terms of a 40-year-old debate about pollution, so as to finally resolve it. However, the fact that economists have applied bilateral marginal cost pricing in some areas—most notably in social choice—but not consistently in others indicates that our contribution is nevertheless timely. We have shown that bilateral marginal cost pricing can provide novel insights into the law of torts, and many additional social interactions arise in which the parties impose costs on each other that are not internalized through market prices. Consistent application of marginal cost pricing will improve our understanding of these situations.

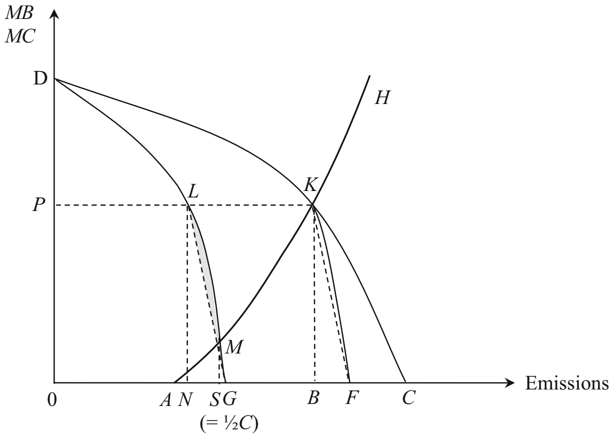
Acknowledgements We thank Glen Weyl for very helpful suggestions and comments on an earlier draft.

Appendix

This appendix confirms that, in Fig. 2, the sum of the areas of the shapes *NLMS* and *SMKB* (= *NLMKB*) equals the area of *GMKF* by construction. Figure 6 describes the same situation as Fig. 2 without emphasizing the shaded areas. Start with rectangle *NLKB* with height *NL* and width *NB*, which consists of the shapes *NLMS*, *SMKB*, and *LKM*. Shift the base of rectangle *NLKB* to the right, thus turning it into a parallelogram whose lower left vertex coincides with point *G*. The lower right vertex of the parallelogram defines point *F* so that the distances *NG* and *BF* are identical. Because the area of a parallelogram is the product of its base and its height, the areas of parallelogram *GLKF* and rectangle *NLKB* are identical.

Now account for the non-linearity of the marginal benefit curve *DC*. Consider the grey shape that is defined by the line *LG* and the arc formed by the part of the marginal benefit curve *DG* that starts at *L*, goes through *M* and ends at *G*. Subtract this grey shape from parallelogram *GLKF*. Add an identical shape at the right edge *KF* of parallelogram *GLKF*,

Fig. 6 The equivalence of the areas *NLMKB* and *GMKF*



thereby defining the solid curve that connects points *K* and *F*.[22] It follows that the area of the shape defined by the four solid lines *GLKF* is the same as the area of parallelogram *GLKF* and thus the same as the area of rectangle *NLKB*.

Finally, note that shape *LKM* is part of the rectangle *NLKB* as well as of the shape defined by the four solid lines *GLKF*. Subtracting *LKM* from both shapes leaves the shapes *NLMKB* and *GMKF*. Thus, the areas of *NLMKB* and *GMKF* are identical.

The area of *AKB* represents the cost of the emissions by the two factories that the resident bears, and the area of *BKC* represents the control cost that the two factories bear. The area of *BKF*, defined by the solid line *KF*, that measures the control cost of the first factory when the second factory operates is the sum of the first factory's control cost *SMG* before the second factory started operating and the area *NLMS*. Hence the area of *FKC* that is defined by the solid lines is the second factory's control cost.[23]

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[22] That is, the solid curve *KF* and the solid curve *LMG* are equidistant at all points.

[23] Had the second factory's presence shifted the marginal benefit curve to the solid curve *DKF* instead of *DKC*, then the second factory would obtain benefits from emitting the amount *BN* (since *K* differs from *M*) but no benefits from any additional emissions. Because the tax *NLMKB* would not provide an incentive for the second factory to reduce emissions below *BN*, there would be no shape *FKC* and the second factory would bear no control cost.

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